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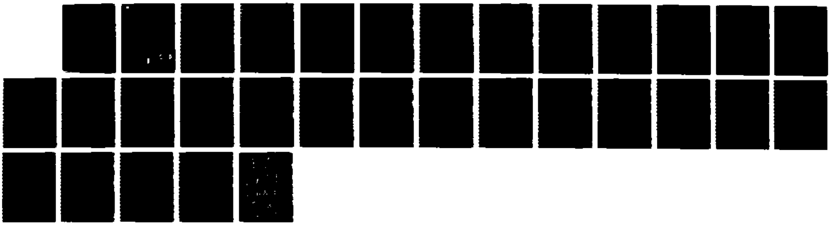

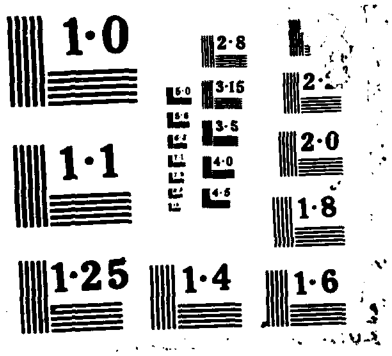
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FAST ALGORITHM DEVELOPMENT FOR LARGE-EDDY SIMULATION OF CIRCULAR-JET TURBULENCE

L. Krishnamurthy, C. A. Hall, and T. A. Porsching

FINAL TECHNICAL REPORT

FOR THE PERIOD 15 SEPTEMBER 1985 THROUGH 14 OCTOBER 1987

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH

CONTRACT NO. F49620-85-C-0137

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UNIVERSITY OF DAYTON
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The research reported herein addresses a single, free, round turbulent jet expanding into a quiescent environment, and deals with the large-scale motions through large-eddy simulation and the small-scale motions through subgrid-scale turbulence modeling. The computational fluid dynamic investigation of the nearfield downstream of an axisymmetric nozzle is based on a two-dimensional procedure involving a variable reduction method, with the farfield fully developed flow structure analyzed by matched asymptotic expansions for large Reynolds numbers. The numerical algorithm development examines vectorization, three-dimensional flowfield, and the construction of weakly dissipative difference methods. Keywords:

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TABLE OF CONTENTS

SECTION I

INTRODUCTION

This final technical report documents the current status and accomplishments of the research performed by the University of Dayton for the Air Force Office of Scientific Research (AFOSR), Fast Algorithms Initiative under Contract No. F49620-85-C-0137. The research documented herein was conducted by the University of Dayton Research Institute (UDRI) as the prime contractor, with the University of Pittsburgh, Institute for Computational Mathematics and Applications (ICMA), as its subcontractor. Although issued as a final report, this document nevertheless presents the results of research performed during the first two years of an effort that was initially anticipated as a three-year program. Since further research remains to be accomplished under Contract F49620-88-C-0040, in response to the AFOSR Broad Agency Announcement, this report is primarily an outline of the research progress to date.

1. BACKGROUND

The traditional viewpoint of fluid mechanical turbulence is that it begins as the onset and growth of hydrodynamic instabilities in an erstwhile laminar flow. According to this view, the energy driving the macroscopic instabilities cascades through higher and higher wavenumbers, with the flowfield culminating in what is commonly referred to as the fully developed turbulence, and characterized by the interaction of a very large number of disparate length and time scales. Such a physical identification of the laminar-to-turbulent transition as a sequence of hydrodynamic instabilities and restructuring of the rather homogeneous initial velocity distribution of the original laminar flow would seem especially appropriate for free-shear layers (which are not subject to the influence of solid boundaries, except at the flow origin). For the single, free round jet issuing out of a circular nozzle into a quiescent

ambient, for example, a surface discontinuity develops in the free shear layer immediately downstream of the nozzle, with a point of inflection in the velocity profile. Under such conditions, the free shear layer is unstable to infinitesimal disturbances at almost any initial jet Reynolds number. This shear-layer instability gives rise to the well known Kelvin-Helmholtz instability waves if the shear layer is initially laminar. With the rolling-up of the initial shear layer, oscillations develop in the jet generating periodic vortex rings. This has been clearly demonstrated by the smoke photograph of Wille (1963) of the ring vortices in the initial region of a circular jet (having a Reynolds number of 70,000 at the jet origin).

In the so-called mixing region downstream of the instability region, the vortex rings interact with each other and spread the vorticity until the central potential-core region (which typically extends for about four initial jet diameters) disappears. The later stages of transition in axisymmetric mixing layers were explored by Wille (1963) to show that the vortex ring disturbances (which are instability eigen modes) could pair at intervals. In the transition region the flow relaxes into its final stages of development. This region is the domain of large-scale structures and is characterized by the strong transfer of the axial jet momentum in the radial direction, i.e., a high shear stress. Finally, in the fully developed turbulent region the jet becomes self-similar.

2. PREDICTION

Increasing attention has been paid to the initial and transition regions of the flowfield, with the hope that a study of the origin, evolution, and interaction of vortex-ring-like structures will reveal some clues about the eventual turbulent state. That was not the case in the past when instability was treated quite separately and differently from turbulence. For instance, the eigenmodes of instability were determined from the

solution of an unsteady linear partial differential equation, the well known Orr-Sommerfeld equation, whereas the Reynolds equations (which are the time-averaged nonlinear Navier-Stokes equations but rendered incomplete due to the so-called closure problem) were considered to provide an adequate description of the fully developed turbulence. It should not be surprising, however, that neither the time-dependent linear formulation, nor the time-averaged nonlinear formulation has led to a viable turbulence theory, since the laminar-to-turbulent transition is an extremely complex process. Although transition does involve an initial inviscid instability mechanism, the molecular viscosity has an influence on both the rolling-up of the shear layer into vortex structures, and the core sizes of the rings and the mode of the subsequent three-dimensional instability. Moreover, viscosity is again found to have a stabilizing influence in the basically inviscid process of coalescence or pairing of the ring vortices. Clearly, an accurate predictive methodology must take into account this dichotomous nature of the flowfield involving the large-scale inviscid aspects and the small-scale viscous aspects, which manifest themselves as partly deterministic and partly stochastic behavior.

That the initial shear-layer instability of the free turbulent jet has a significant influence on its subsequent development implies a strong coupling between the initial instability and the eventual turbulent state. Thus, the prediction of turbulence cannot be approached in isolation via the Orr-Sommerfeld equation. The conventional point of departure in turbulence-prediction computations has been the Reynolds-averaged Navier-Stokes equations. Although these equations, unlike the inviscid formulation, do not neglect any terms in the full equations, they still suffer from the indeterminacy directly arising from the averaging procedure. The Reynolds equations are obtained by averaging the Navier-Stokes equations over a time interval which is large compared to the characteristic times of the turbulent eddy fluctuations but small compared to the macroscopic flow changes. In such an averaging procedure, the

nonlinear convective terms give rise to various new terms which, in the absence of sufficient equations, need to be modeled. The principal model inaccuracy, irrespective of the level of closure, lies in the fact that it is done for all wavenumbers.

A logical predictive approach stemming from the traditional viewpoint is to seek the evolution of jet turbulence by solving the time-dependent Navier-Stokes equations. The exact solutions of these equations for the entire range of the time and length scales prevailing in a jet would then explicitly describe and predict the flowfield. Unfortunately, a typical turbulent jet is characterized by a large number of randomly interacting length scales which range from those as large as the local diameter of the jet to ones as small as the Kolmogorov dissipative scales. The number of grid points N needed by a computational grid to resolve the smallest scales is of the order of $Re^{9/4}$, where Re is the local Reynolds number (see, e.g., Case et al. 1973). The estimated values of N for a realistic turbulent jet are extremely large, on the order of 10^{13} and higher (Chapman 1979). The resulting enormous computing requirements imposed by the time-dependent, multidimensional Navier-Stokes equations in simulating a realistic jet flow are well beyond the capabilities of today's largest and fastest computers. Indeed, even with the computers of the foreseeable future, the smallest scale explicitly resolved is very much larger than the dissipation scale. In other words, accurate solutions of the full three-dimensional, time-dependent equations are possible at present only for the pre-transition flows at very low Reynolds numbers.

Thus, the only feasible approach for jet-turbulence prediction at present (and in the foreseeable future) involves the direct computation of the complete equations on a coarsely resolved grid (dictated by the available computing resources) through the large-eddy simulation (LES) for describing the "large"-scale motions, the development of model(s) to account for the "small" subgrid-scale (SGS) turbulent mixing, and the proper

coupling of the SGS modeling to the LES computations. Such an approach also ensures that whereas the large-scale motions are properly treated deterministically in LES, the SGS turbulence can be treated statistically. What is interesting to note here is that unlike the computation with the Reynolds-averaged equations which necessitates modeling at all wavenumbers, the SGS turbulence modeling is needed only for wavenumbers higher than the LES cutoff wavenumber (which corresponds to π/Δ_f where Δ_f is the LES filter width). Furthermore, even relatively crude modeling of the subgrid-scale motions is tolerated (in contrast to the modeling approaches for the Reynolds-averaged equations), since the large-scale dynamics is essentially insensitive to inaccuracies of SGS modeling, so long as the latter is dissipative, i.e., it is guaranteed to remove the excess energy from the resolved scales in LES (see Paragraph II.1).

Finally, the prospects for accurate and economical prediction of the turbulent round jet through LES computations and SGS modeling are greatly enhanced at present. The factors contributing to this possibility are the increasing availability of vector computers of large size and speed, the potential for significant improvements in the development of efficient numerical algorithms, and the considerable scope for efficient matching of these algorithms to emerging computer architectures.

3. SCOPE AND OBJECTIVES

The overall theme of the UDRI-ICMA joint research program is the efficient computation of the development of the free turbulent round jet by time-dependent Navier-Stokes equations. This research encompasses the LES computation on a coarsely resolved grid, the development of accurate SGS models, and the proper coupling of the SGS turbulence to the LES computations. As discussed in Krishnamurthy et al. (1986), the large-scale motions are computed by the ALgorithms for GAs Equations (ALGAE) code, developed at ICMA. This computer code, currently limited to two-dimensional geometry, does not produce a true LES of the

circular-jet turbulence (which remains axisymmetric only in the mean). Nevertheless, our research has employed ALGAE as a baseline procedure for LES, with the necessary refinements and optimizations implemented through successive modifications.

The scope of the UDRI-ICMA research during the past two years can be summarized by the following objectives:

- Examination of LES techniques for application to the circular-jet problem.
- Investigation of SGS turbulence modeling procedures for prescribing appropriate models to describe the small-scale motions in the development of a free jet.
- Investigation of the use of coupled stable and unstable difference schemes to reduce numerical dissipation and promote sustained quasi-periodic motion.
- Study of the applicability of conservation principles to the construction of Total Variation Diminishing Schemes.
- Examination of techniques to reduce the computational effort involved in the solution of discrete three-dimensional fluid models.
- Examination of the underlying logic of the ALGAE procedure to identify those portions of the code that are amenable to vectorization.
- Incorporation into ALGAE (or its successors) of new SGS turbulence models for jet flows as they become available.

Our research to date has entailed an analysis and assessment of appropriate LES and SGS modeling with the view to

arriving at suitable schemes for investigating the round-jet flowfield. A review of these aspects is available in Krishnamurthy et al. (1986). As noted therein, this research program has involved several interdependent tasks. Some of these taks are independently conducted by UDRI (e.g., SGS turbulence modeling and asymptotic analyses of the flowfield behavior) and by ICMA (e.g., ALGAE modifications) and some have entailed joint efforts (e.g., boundary-condition modifications, and incorporation and testing of an algebraic eddy-viscosity model). The salient features of these investigations are discussed in the next section.

4. OUTLINE OF REPORT

The current status of the research and its accomplishments are given in Section II. Section III lists the documentation from the research sponsored under this program. The UDRI and ICMA personnel supported by this program are shown in Section IV.

SECTION II

STATUS OF RESEARCH

A brief summary of the progress in achieving the research objectives outlined in the previous section and of the accomplishments to date is presented below.

1. EXAMINATION OF LES TECHNIQUES

The essence of LES lies in the description of the circular-jet turbulence without requiring the solution of the full time-dependent Navier-Stokes equations for all the scales, while retaining the essential physics. At the high Reynolds numbers typical of a turbulent jet, explicit resolution by LES is limited to turbulence length scales larger than a selected cutoff length (which is the filter width Δ_f used in obtaining the LES equations). The scales that remain unresolved range from the LES cutoff scale to the Kolmogorov scale. Since they are discarded in LES, a closure approximation is needed to model their influence on the explicitly resolved scales. The LES cutoff, to be sure, is arbitrary and is also a function of the current status of computer resources. Although the fraction of the spectrum of the energy containing turbulent eddies (length scales) that can be explicitly resolved can be expected to grow toward unity as computers become larger and faster, the LES cutoff for high-Reynolds-number realistic flows is unlikely to approach the order of the Kolmogorov scale. This has a major implication for LES predictions.

When the LES computations involve the original Navier-Stokes equations (without any modification of the convective terms therein) and an arbitrary cutoff in the resolution (as encountered, for example, in the ALGAE code), the computed solutions lead to physically unacceptable flowfields. Present research has identified the accumulation of unrealistic excess energy at the computed scales as the major source of this

difficulty. This becomes clear when it is recognized that any energy-conserving numerical algorithm collects energy at the smallest resolved scales (which are in the vicinity of the LES cutoff), until an equilibrium is established between the cascade and dissipation rates. The trapping of this excess energy at the mesh scale (in the absence of a proper mechanism for its cascading to the Kolmogorov scale) causes an unrealistically large transfer of energy from the large scales. This conclusion is readily apparent, for instance, when the influence of the unresolved (and discarded) scales on the LES-computed scales is modeled by an eddy viscosity which is proportional to the product of the length and velocity scales of the trapped energy. As this energy increases, the eddy viscosity increases and the energy transfer from the computed scales grows unrealistically high.

Present investigations have examined several modifications to the convective terms with the objective to uncover a mechanism for the removal of energy from the computed scales that mimics the physical cascade process as closely as possible. A realistic LES of the round jet, therefore, requires suitable modifications of the governing equations in the ALGAE procedure. The appropriate modifications to the convective terms in the ALGAE code remain to be implemented and validated.

2. INVESTIGATION OF SGS MODELING

The closure models for SGS turbulence that have been attempted to date are either one-point or two-point models (as characterized by the number of spatial points appearing in the statistics). The latter are far more complicated and have been limited so far to homogeneous (and usually isotropic) flows. For instance, Chollet (1983) has shown that the two-point closure based upon the Eddy Damped Quasi-Normal Markovian (E.D.Q.N.M.) approximation is able to represent statistically the distribution of energy among the various scales and the interaction between the large and the small scales. A further application of the E.D.Q.N.M. closure by Bertoglio and Mathieu (1983) to an

anisotropic case, viz., homogeneous turbulence subjected to a uniform shear, has revealed some major implications for the eddy-viscosity model. It was noted earlier that this model ensures a drain of the excess energy from the resolved scales to the subgrid scales. An acceptable SGS turbulence model, in addition, must be able to predict a return of energy (denoted as the input term in Kraichnan (1976) and the backscatter in Leslie and Quarini (1979)) from the subgrid scales to the large eddies. The reults of Bertoglio and Mathieu, however, suggest that the eddy-viscosity model is inadequate in that regard and imply that new "de-averaging" models may be required to account for the return of energy when a turbulent shear flow is addressed. This aspect would have a critical bearing on round-jet prediction, in view of the fact that an eddy which is subgrid somewhere (e.g., sufficiently far downstream of the nozzle) needs to be explicitly resolved elsewhere (where the grid is finer), since that signifies the re-emergence of a structure from the subgrid model into a region of the grid where it can be resolved.

Present research has focused on a one-point closure model, with the choice being an eddy-viscosity model. This model assumes that the SGS Reynolds stress deviator tensor, T_{ij}, is proportional to the local strain-rate tensor of the filtered LES field, S_{ij}:

$$T_{ij} = -2\ \nu_t S_{ij}.$$

Here, ν_t is the eddy viscosity which is assumed to be proportional to the product of the appropriate SGS length and velocity scales; i.e.,

$$\nu_t \propto \Delta_f q.$$

Note that the use of the filter width, Δ_f, for the characteristic length scale of SGS turbulence is reasonable because it corresponds to the largest (and presumably the most important) unresolved eddy of SGS turbulence.

A popular eddy-viscosity model in LES research has been the Smagorinsky model (see, e.g., Smagorinsky (1963) and Mansour et al. (1978)). The choice in the present research, however, is based on the algebraic mixing-length formulation due to Launder et al. (1972), and is largely motivated by the need to incorporate and test a variable-viscosity capability in the ALGAE code. Such a capability has been implemented in that code (see Paragraph II.3); a successful validation of the round-jet calculations including the effects of variable eddy viscosity would be helpful in providing benchmark data for comparison with more refined SGS models.

An important aspect of the present investigations of SGS turbulence deals with the asymptotic analysis of the farfield behavior. It is well known that both free and wall mixing layers in turbulent shear flows asymptotically reach the so-called fully developed state when the flow becomes self-similar. The recently completed analysis of Bush and Krishnamurthy (1987) on the adverse-pressure-gradient boundary layer has demonstrated the viability of the mixing-length model. Although these results do not apply directly to the present consideration of a single free jet, they are relevant to a ducted jet of possible LES effort in the future. Furthermore, the problem formulation and analysis of the three-layer asymptotics of the previous study have led to the ongoing asymptotic analysis of Bush and Krishnamurthy (1988) for the fully developed region of the round jet. The latter is expected to be directly relevant to the examination of the eddy-viscosity model for the SGS turbulence. In addition, the round-jet analysis furnishes the farfield pressure distribution with which the boundary conditions imposed by the ALGAE computations on certain artificially introduced psuedo boundaries must be consistent (see Paragraph II.5).

3. ALGAE MODIFICATIONS

A variable-eddy-viscosity capability has been added to ALGAE. This is based on the algebraic mixing-length model of

Launder et al. (1972). The computational testing of this capability awaits the determination of the optimal boundary conditions (see Paragraph II.5).

For the two-dimensional version of the code, the maximum number of allowable mesh points has been increased from 4800 to 6000.

The vectorization effort of ICMA includes the conversion of the code for operation on the CRAY-XMP at the NSF Pittsburgh Supercomputer Center and the use of solid-state disk reads and writes during the frontal solution of the dual-variable system.

The software necessary to make 16mm movies of computer-generated transient velocity and pressure fields has been developed and implemented on the ICMA Datagraphics communications unit. Furthermore, ICMA has recently acquired a state-of-the-art IRIS color graphics workstation and animation peripherals. Installation of the software necessary to transfer this movie-making capability to the IRIS is in progress.

4. THREE-DIMENSIONAL COMPUTER CODE

Specification of a Dual-Variable solution of the discretized three-dimensional fluid dynamics equations is underway. In particular, a method for generating the required three-dimensional cycle basis has been developed by Ye (1987). Moreover, the decision has been made to utilize (block) iterative, rather than direct, methods in the solution of the dual-variable system. This decision is based on empirical studies (Hageman (1975a,b), Mesina (1987)) indicating the efficiency of iterative methods in the solution of large-scale systems, such as those arising from the discretization of three-dimensional transport models.

5. FARFIELD BOUNDARY CONDITIONS

The specification of boundary conditions remains a key issue in the numerical simulation of the turbulent jet by ALGAE and is expected to govern the success of such a simulation. Figure 1 shows the geometry of the model jet. Note that the boundary condition at the axis of symmetry does not pose any difficulty. It is the farfield boundaries (at large radial distance and far downstream) that cause problems. For arriving at the optimal farfield conditions, Porsching (1986a) has considered a model jet problem, viz., the two-dimensional, inviscid jet. A calculation procedure has been developed at ICMA to compute the implicitly defined analytic solution of this flowfield. This procedure has the capability to (a) determine the streamlines intersecting the ALGAE-grid mesh points, (b) compute the analytic velocity and pressure fields at those mesh points, and (c) convert these data to ALGAE boundary conditions.

Figure 2 shows the streamlines of the analytic solution, while Figures 3 and 4 present respectively the ALGAE numerical solutions obtained on a 28 x 35 grid with an inviscid-flow model and with a model incorporating a constant molecular viscosity of 0.02. Both solutions clearly reproduce the qualitative features of the analytic solution. The inviscid ALGAE numerical solution shows a tendency towards recirculation in the lower right-hand side of the flow region even though it is the inviscid model that gives rise to the analytic solution. This is due to a slight inaccuracy in the resolution of the jet's boundary layer where it meets the wall. Insufficient resolution of this boundary layer leads to numerical solutions that are completely inaccurate (Figure 5). The viscous ALGAE numerical solution, on the other hand, shows no recirculation tendencies.

As noted in Paragraph II.2, another approach to obtain the farfield pressure distribution is through an asymptotic analysis of the fully developed region. Whereas the LES research addresses the development of a jet discharging into an unbounded

Figure 1.
Problem Geometry

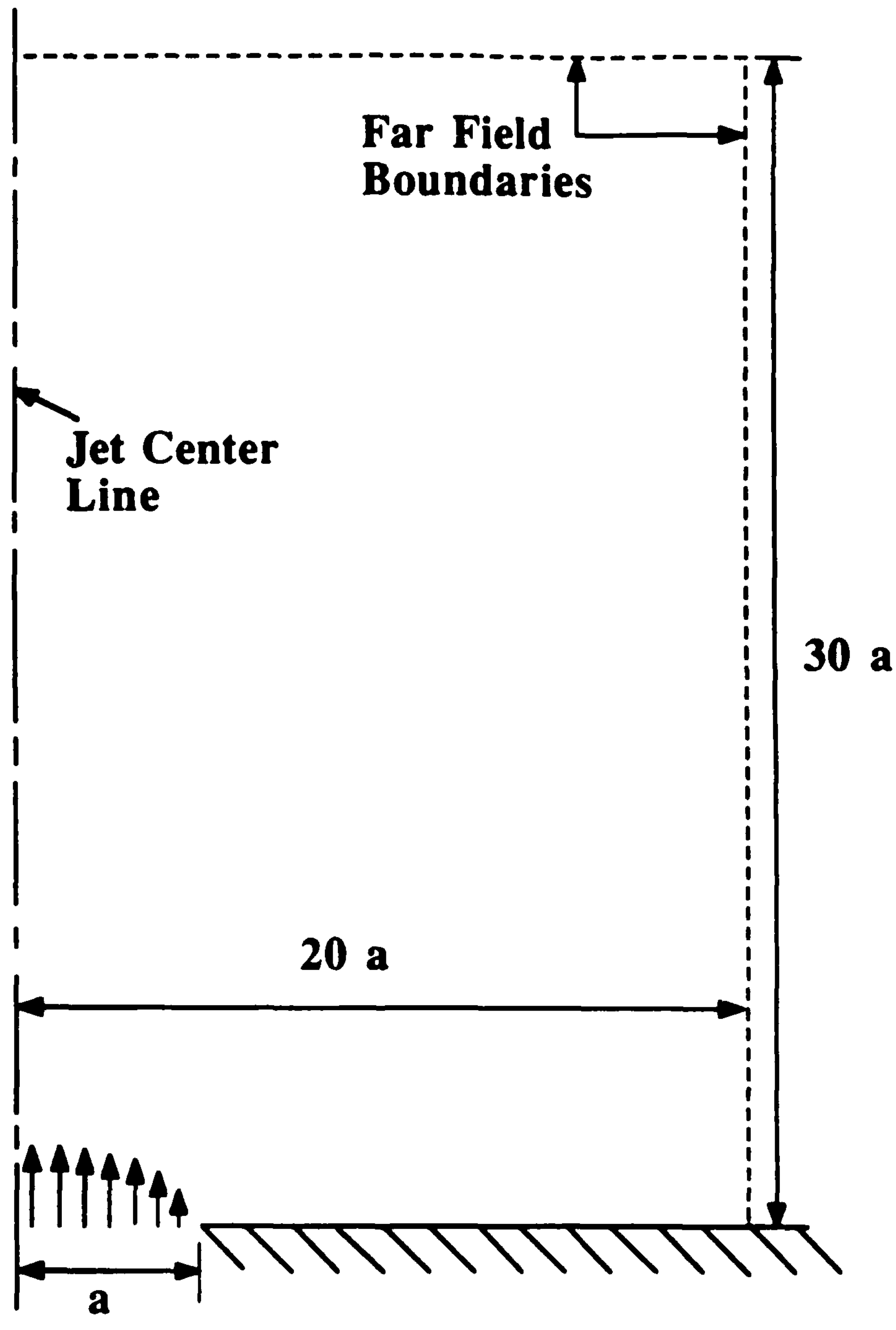


Figure 2.
Analytic Solution Streamlines
(Kirchhoff 1869)

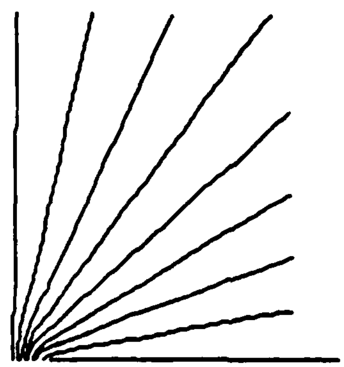


Figure 3.
ALGAE Solution Streamlines
Inviscid Flow

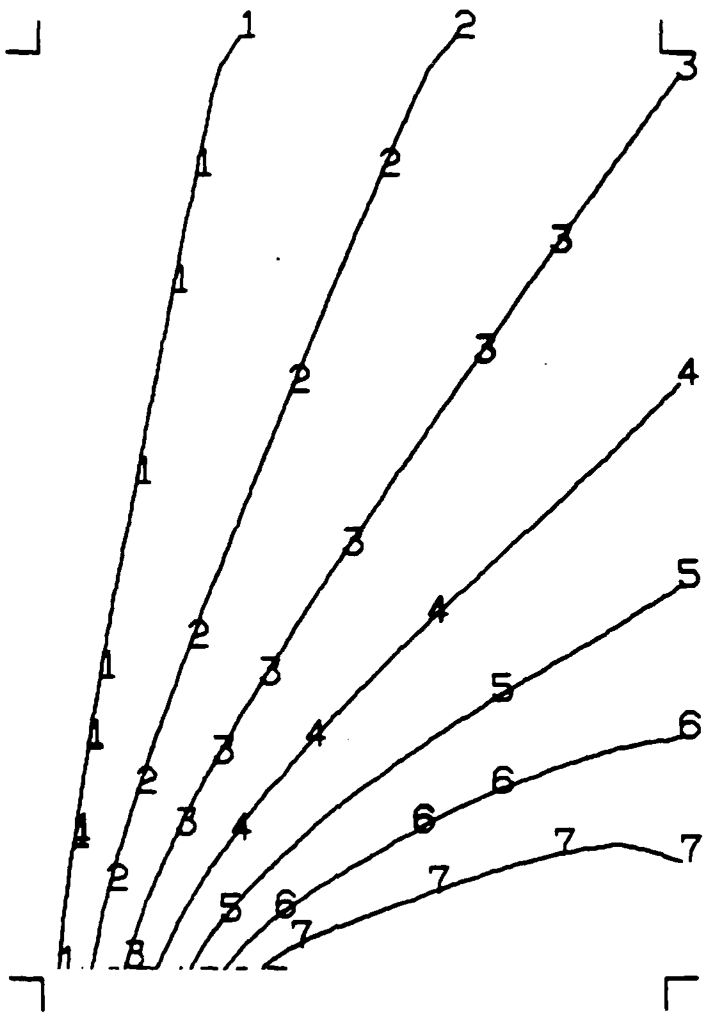


Figure 4.
ALGAE Solution Streamlines
Molecular Viscosity = .02

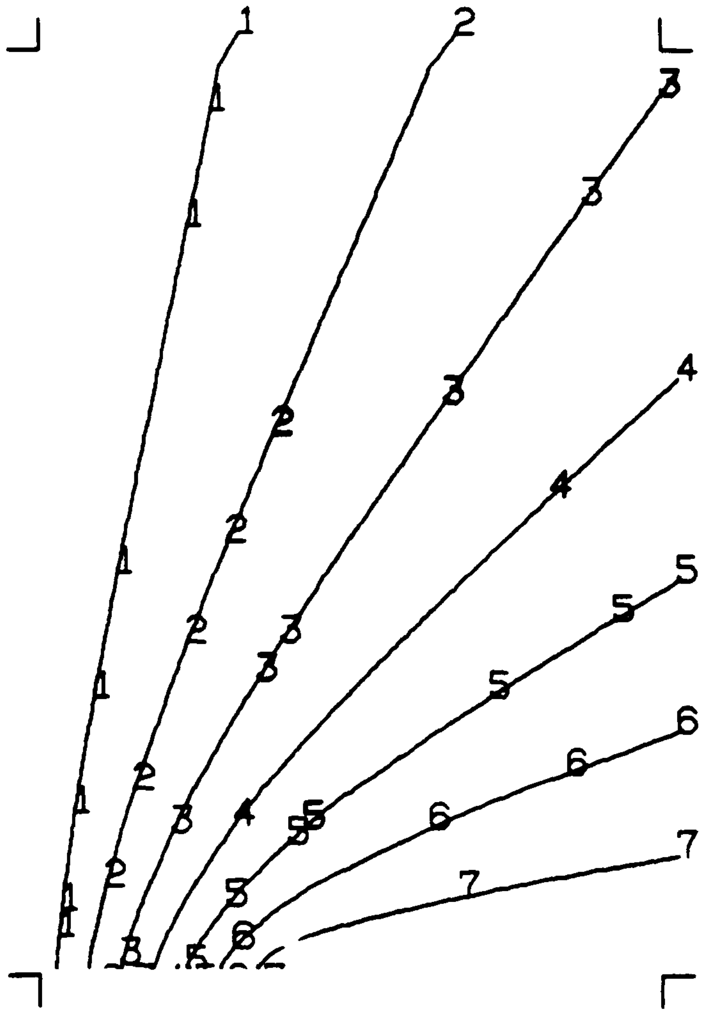
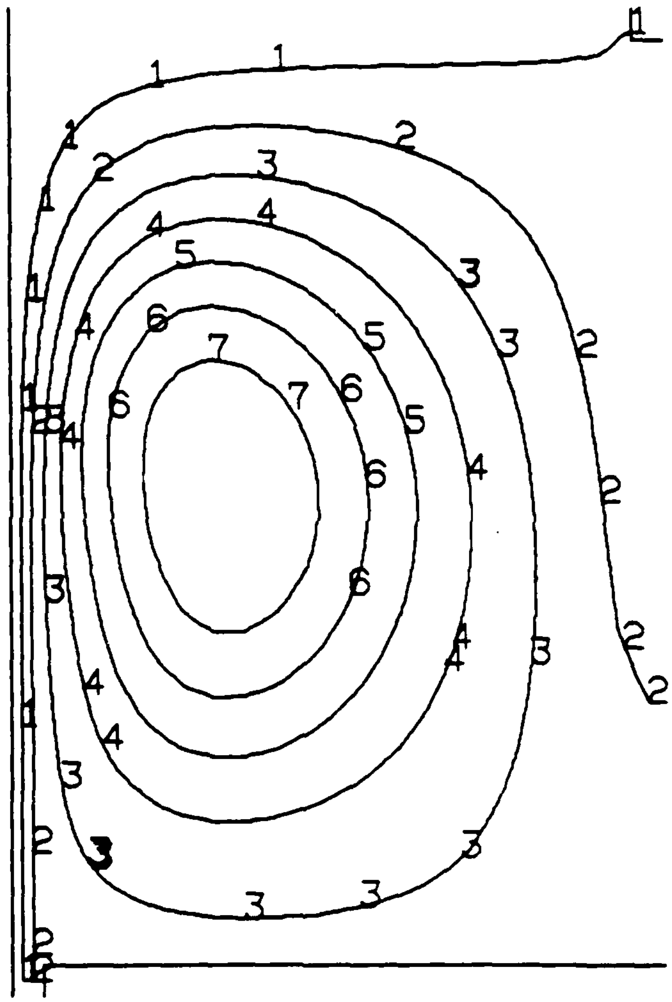


Figure 5.
ALGAE Solution Streamlines
Insufficient Boundary Layer Resolution



domain, the ALGAE-based computation requires confined flowfields and introduces artificial pseudo boundaries. The asymptotic farfield solutions do apply to the downstream pseudo boundary, provided it is at least 10 jet diameters downstream of the nozzle (experimental evidence suggests that the fully developed region where self-similarity holds occurs between 10 and 50 jet diameters). They are not valid, however, at the radially outward boundary (i.e., the top boundary where free-slip wall conditions have been imposed in the ALGAE computations) for distances up to 10 jet diameters.

The asymptotic results in Bush and Krishnamurthy (1988) are found to be consistent with the experimental results of Reichardt (1942) and the theoretical results of Gortler (1942). Thus, to the leading order of approximation, the downstream variation of the centerline axial velocity is given by

$$u_c = 2\ (6.58)/x,$$

where x is the nondimensional axial distance from the jet origin. The radial distribution of the axial velocity, u, is given by

$$u/u_c = 1/(1+c^2\eta^2)^2,$$

where $c = 1/\sqrt{3}$, $\eta = (r/x)/\delta$, $\delta = 0.076$, and r is the nondimensional radial distance from the axis of symmetry. The farfield nondimensional excess pressure (dimensional pressure in excess of the ambient pressure, normalized by the initial jet momentum) of the jet, to the leading order of approximation, is

$$p = \Pi_0(\eta;\delta)/\delta x^2,$$

where Π_0 is determined from the zeroth-order solution to be

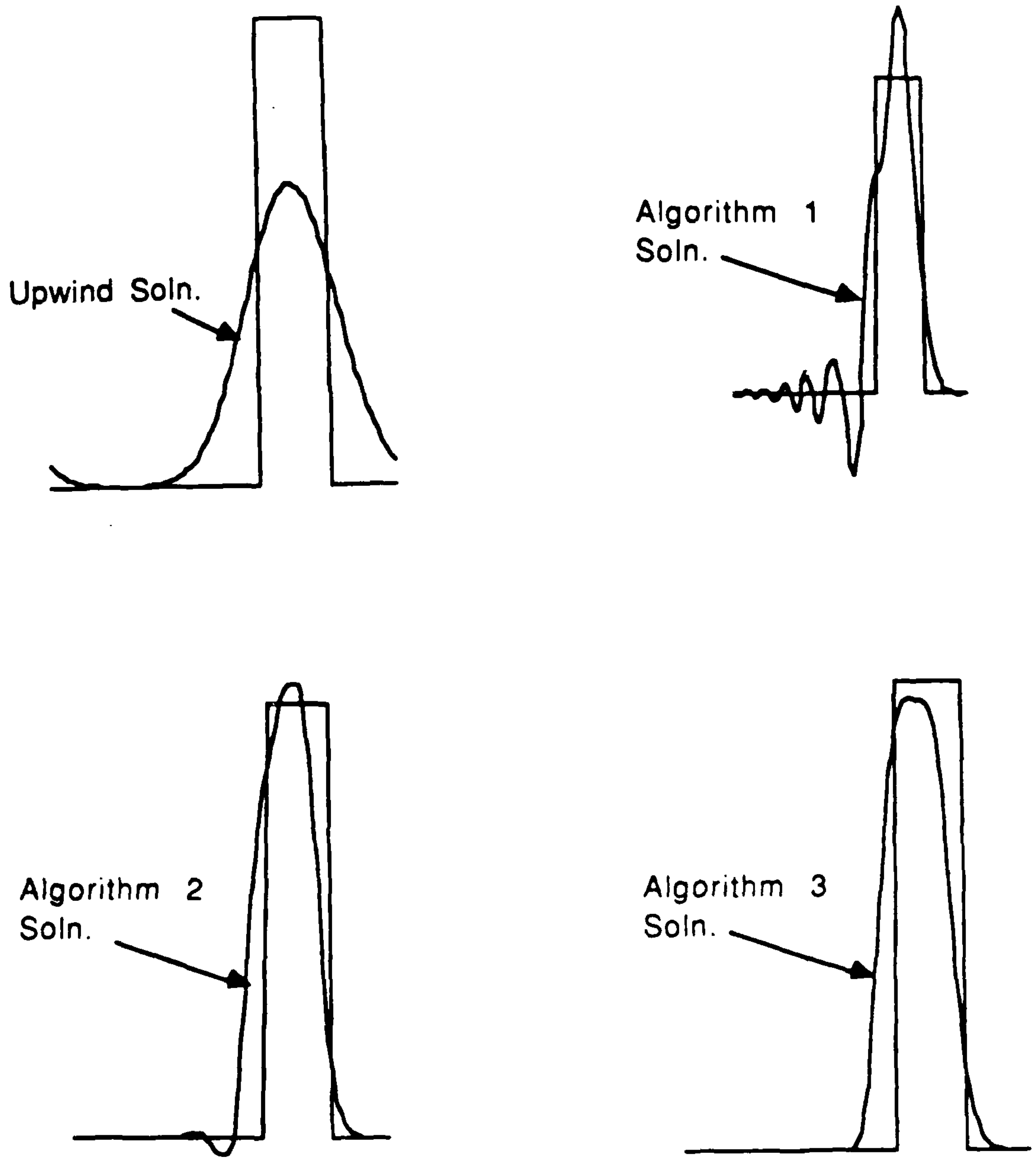
$$\Pi_0 = \lambda_0(1-3k)/(1+k)^3,$$

with $k = c\eta$ and $\lambda_0 > 0$. It is interesting to note that the foregoing results imply that Π_0 and hence the excess pressure is positive for $0 \leq k < 1/3$ and is negative for $1/3 < k < O$. Thus, the farfield jet pressure exceeds the ambient pressure for $0 \leq r/x < 4.39 \times 10^{-2}$ and is less than the ambient pressure for $4.39 \times 10^{-2} < r/x < O$. It is clear that the ALGAE-based predictions must be consistent with the asymptotic results for axial distances exceeding 10 jet diameters.

6. WEAKLY DISSIPATIVE DIFFERENCE METHODS

The idea of using discrete energy-conservation principles to construct weakly dissipative hybrid difference methods has been developed for a class of linear convection equations in Porsching (1986b). This work contains several algorithms for the determination of the weights that appear in such schemes. Three of these algorithms, labeled Algorithms 1, 2, and 3 in increasing order of sophistication, have been tested numerically on the problem of pure convection of a square wave. The results of these tests are shown in graphic form in Figure 6. For comparison, the well known "upwind" solution (which is prone to excessive numerical diffusion) is also included. Numerical solutions are shown superimposed on the exact solution. It is obvious that all three algorithms dramatically reduce the numerical diffusion of the upwind scheme. Moreover, the dispersive "ripples" of Algorithm 1 are substantially attenuated by Algorithm 2, and completely eliminated by Algorithm 3.

Figure 6.
Hybrid Solutions



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SECTION III

DOCUMENTATION

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Bush, W. B. and Krishnamurthy, L. (1987) "Asymptotic Analysis of a Turbulent Boundary Layer in a Strong Adverse Pressure Gradient," UDR-TR-87-35, Submitted for Publication.

Bush, W. B. and Krishnamurthy, L. (1988) "Asymptotic Analysis of the Fully Developed Region of an Incompressible, Free, Turbulent, Round Jet," in preparation.

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Porsching, T. A. (1986) "On Weight Selection Procedure for Hybrid Difference Methods," Technical Report, ICMA-86-103.

SECTION IV

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